

Supporting Information for:
Single Molecule Rectification Induced by the
Asymmetry of a Single Frontier Orbital

Wendu Ding, Christian F. A. Negre,^{*} Leslie Vogt, and Victor S. Batista^{*}

*Department of Chemistry, Yale University, P.O. Box 208107, New Haven, CT 06520-8107, and
Energy Sciences Institute, Yale University, P.O. Box 27394, West Haven, CT 06516-7394*

E-mail: christian.negre@yale.edu; victor.batista@yale.edu

^{*}To whom correspondence should be addressed

Transmission Function: Tight-Binding Model

This section provides a derivation of Eq. (1) of the main text, giving an expression for the transmission function as a function of the applied bias potential for a tight binding model with a single transport channel. The Green's function for such a molecule in between the electron source and drain reservoirs (Figure 1) is

$$\mathcal{G}_{dev}(\varepsilon) = [\varepsilon - H_{dev} - \Sigma_T(\varepsilon)]^{-1} \quad (1)$$

where ε is the energy, H_{dev} is the molecular Hamiltonian, and Σ_T is the total self-energy of the contacts, which can be decomposed as the sum of the self-energy of the left and right lead contributions: $\Sigma_T(\varepsilon) = \Sigma_L(\varepsilon) + \Sigma_R(\varepsilon)$.

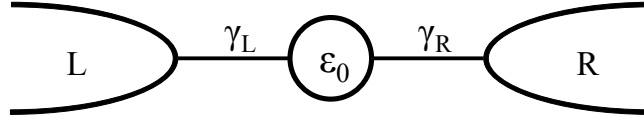


Figure 1: Scheme of a molecule with a single transport channel state $|\varepsilon_0\rangle$, coupled to the left (L) and right (R) electron reservoirs with coupling constants γ_L and γ_R , respectively.

The retarded and advanced Green's functions are

$$\mathcal{G}_{dev}^+(\varepsilon) = \frac{1}{\varepsilon - H_{dev} - \Sigma_T(\varepsilon)} \quad (2)$$

$$\mathcal{G}_{dev}^-(\varepsilon) = \frac{1}{\varepsilon - H_{dev} - \Sigma_T^\dagger(\varepsilon)} \quad (3)$$

The self-energy for the left (L) and right (R) contacts are

$$\Sigma_{L/R}(\varepsilon) = \Delta_{L/R} + i\Gamma_{L/R}(\varepsilon) \quad (4)$$

where the real part Δ can be approximated as $\Delta_L \simeq \frac{V}{2}$ and $\Delta_R \simeq -\frac{V}{2}$, with V the applied bias voltage.

The total self-energy is

$$\Sigma_T(\varepsilon) \simeq i[\Gamma_L(\varepsilon) + \Gamma_R(\varepsilon)] \quad (5)$$

Considering there is only one state $|\varepsilon_0\rangle$ between the two contacts, we have $\langle\varepsilon_0|H_{dev}|\varepsilon_0\rangle = \varepsilon_0$, and the retarded and advanced Green's function take the form:

$$\mathcal{G}_{dev}^+(\varepsilon) = \frac{1}{\varepsilon - \varepsilon_0 - i[\Gamma_L(\varepsilon) + \Gamma_R(\varepsilon)]} \quad (6)$$

$$\mathcal{G}_{dev}^-(\varepsilon) = \frac{1}{\varepsilon - \varepsilon_0 + i[\Gamma_L(\varepsilon) + \Gamma_R(\varepsilon)]} \quad (7)$$

For either the left or right contact:

$$\Sigma(\varepsilon)_{L/R} = \gamma_{L/R}^\dagger \mathcal{G}_{L/R}^+ \gamma_{L/R} = \gamma_{L/R}^2 \mathcal{G}_{L/R}^+ \quad (8)$$

where $\gamma_{L/R}$ is the coupling between the contact and the device, and $\mathcal{G}_{L/R}^+$ is the retarded Green's function of the contact. The second equality results from the fact that $\gamma_{L/R}$ and $\mathcal{G}_{L/R}^+$ commute in this particular case. Together with the expression $\Sigma(\varepsilon)_{L/R} = \Delta_{L/R} + i\Gamma_{L/R}(\varepsilon)$, we have:

$$\begin{aligned} \Delta_{L/R} + i\Gamma(\varepsilon)_{L/R} &= \gamma_{L/R}^2 \mathcal{G}_{L/R}^+(\varepsilon) \\ \implies \mathcal{G}_{L/R}^+(\varepsilon) &= \frac{\Delta_{L/R}}{\gamma_{L/R}^2} + i \frac{\Gamma_{L/R}(\varepsilon)}{\gamma_{L/R}^2} \end{aligned} \quad (9)$$

The density of states (DOS) of the contact can be expressed by the imaginary part of the retarded Green's function of the contact:^{1,2}

$$DOS(\varepsilon) = -\frac{1}{\pi} \text{Im}[\mathcal{G}^+(\varepsilon)] \quad (10)$$

therefore, we can write the imaginary part of the self-energy $\Gamma_{L/R}$ in terms of the contact DOS:

$$\begin{aligned} DOS_{L/R}(\varepsilon) &= -\frac{1}{\pi} \text{Im}[\mathcal{G}_{L/R}^+(\varepsilon)] = -\frac{1}{\pi} \frac{\Gamma(\varepsilon)_{L/R}}{\gamma_{L/R}^2} \\ \implies \Gamma(\varepsilon)_{L/R} &= -\pi \gamma_{L/R}^2 DOS_{L/R}(\varepsilon) \end{aligned} \quad (11)$$

Also,

$$\mathcal{A}_{L/R}(\varepsilon) = i[\Sigma_{L/R}(\varepsilon) - \Sigma_{L/R}^\dagger(\varepsilon)] = -2\Gamma_{L/R}(\varepsilon) \quad (12)$$

Therefore, the transmission function can be computed as:

$$\begin{aligned} T(\varepsilon) &= \mathcal{A}_L(\varepsilon)\mathcal{G}_{dev}^+(\varepsilon)\mathcal{A}_R(\varepsilon)\mathcal{G}_{dev}^-(\varepsilon) \\ &= 2\Gamma_L(\varepsilon)\mathcal{G}_{dev}^+(\varepsilon)2\Gamma_R(\varepsilon)\mathcal{G}_{dev}^-(\varepsilon) \\ &= \frac{4\Gamma_L(\varepsilon)\Gamma_R(\varepsilon)}{(\varepsilon - \varepsilon_0)^2 + [\Gamma_L(\varepsilon) + \Gamma_R(\varepsilon)]^2} \\ &= \frac{4\pi^2\gamma_L^2DOS_L(\varepsilon)\gamma_R^2DOS_R(\varepsilon)}{(\varepsilon - \varepsilon_0)^2 + \pi^2[\gamma_L^2DOS_L(\varepsilon) + \gamma_R^2DOS_R(\varepsilon)]^2} \end{aligned} \quad (13)$$

Under a voltage bias V , we have:

$$T(\varepsilon) = \frac{4\pi^2\gamma_L^2DOS_L(\varepsilon - \frac{V}{2})\gamma_R^2DOS_R(\varepsilon + \frac{V}{2})}{(\varepsilon - \varepsilon_0)^2 + \pi^2[\gamma_L^2DOS_L(\varepsilon - \frac{V}{2}) + \gamma_R^2DOS_R(\varepsilon + \frac{V}{2})]^2} \quad (14)$$

Additional figures

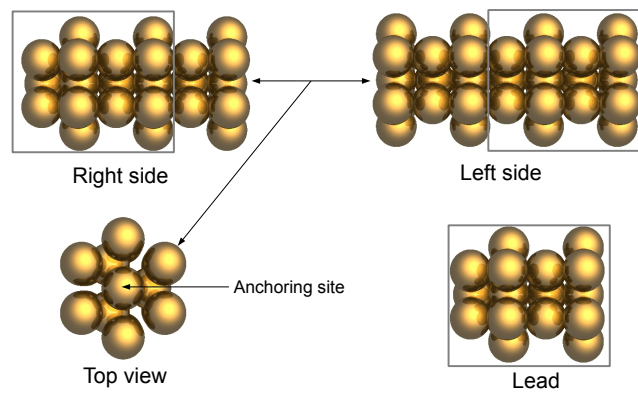


Figure 2: Leads used for building the extended systems described in the text. The Au lattice constant of the hcp lattice is 4.080 \AA . The face of the leads is the (111) surface.

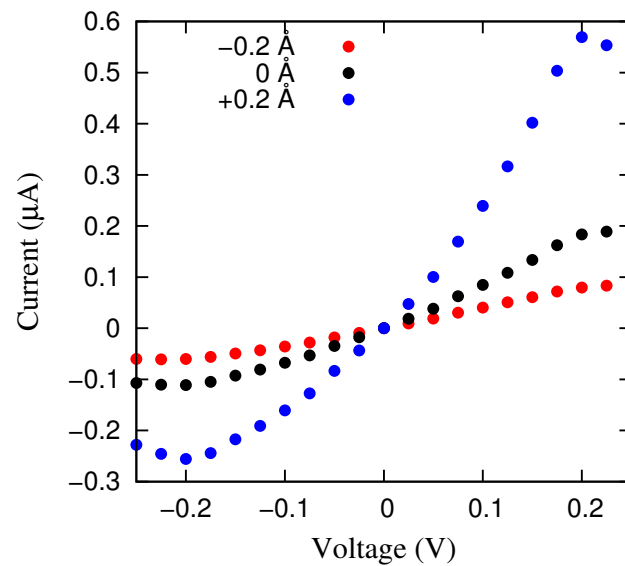


Figure 3: I-V curves for $\text{Au}_L\text{-S-Ph-amide-Ph-S-Au}_R$ for various different S-Au distances.

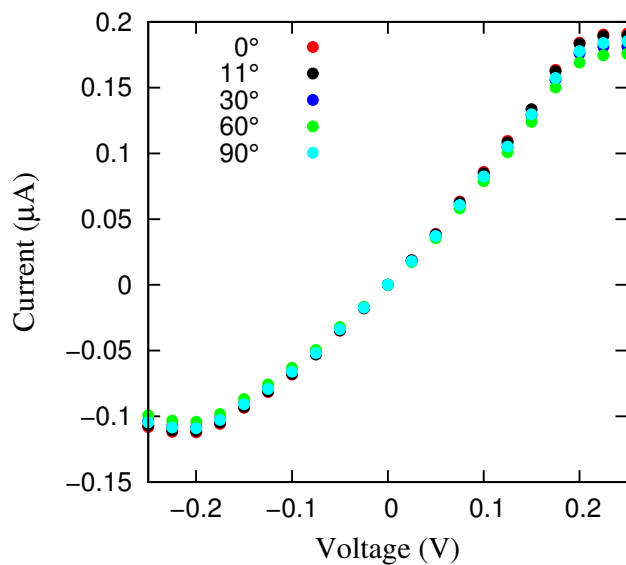


Figure 4: I-V curves for $\text{Au}_L\text{-S-Ph-amide-Ph-S-Au}_R$ for various rotational orientations of the molecule with respect to the contact.

Coordinates of isolated studied systems

HS-Ph-amide-Ph-SH

C	-4.67450400	-0.20420200	-0.02467400
C	-3.94683100	-1.37807200	-0.25661100
C	-3.97529100	0.98643100	0.19883800
C	-2.55787700	-1.35169100	-0.26369100
H	-4.46321200	-2.31750500	-0.42910000
C	-2.58231400	1.01979600	0.19793000
H	-4.51604700	1.91156900	0.37587200
C	-1.85676200	-0.15745000	-0.03641000
H	-2.01053000	-2.27430200	-0.44378100
H	-2.05262700	1.94605300	0.36696900
N	-0.44817600	-0.21925000	-0.04407200
H	-0.06535500	-1.12745300	-0.26018600
C	0.46070200	0.80995600	0.07599500
O	0.14115700	1.99055500	0.17789900
C	1.90237500	0.39152300	0.05640800
C	2.84562000	1.36365600	-0.30164600
C	2.36007000	-0.88709700	0.40603100
C	4.20176300	1.06435900	-0.34469600
H	2.49064000	2.35922700	-0.54465400
C	3.71692900	-1.19356900	0.37559500
H	1.66776300	-1.65122300	0.74868600
C	4.65036000	-0.22121900	-0.00911500

H	4.91104300	1.83270500	-0.63736800
H	4.05145800	-2.18570700	0.66308500
S	-6.46079400	-0.30232900	-0.04553100
S	6.37123600	-0.68625000	-0.03993900
H	-6.69517500	0.96005400	0.36047800
H	6.85750900	0.51962300	-0.39088700

HS-stilbene-SH optimized structure

C	0.47890500	-0.47538500	0.00345300
C	1.92731500	-0.26994300	0.00606600
C	2.77580700	-1.38920900	-0.06945900
C	2.54100100	0.99647700	0.08251800
C	3.92175800	1.13386600	0.07667900
C	-1.92731100	0.27005100	0.00583500
C	-2.77637100	1.38995500	-0.07054300
C	-4.16061200	1.26307600	-0.07673900
C	-4.75058900	-0.00475600	-0.00324500
C	-3.92248000	-1.13392000	0.07803800
C	-2.54059100	-0.99547400	0.08304500
H	-4.78271700	2.15114300	-0.13898500
H	-4.35793200	-2.12702500	0.14100800
H	-1.93177000	-1.89117300	0.15416800
C	-0.47878900	0.47533900	0.00353700
C	4.16138900	-1.26321800	-0.07487800
C	4.75064400	0.00377100	-0.00307100
H	4.36185600	2.12504400	0.13643200
H	1.93187400	1.89195400	0.15218400
H	4.77942600	-2.15431300	-0.13464900
H	2.33757900	-2.38228500	-0.12724900
H	-2.33775500	2.38270200	-0.12923900
H	0.17707900	-1.52141000	-0.00721500
H	-0.17694600	1.52127300	-0.00666500
S	6.51738200	0.26104500	-0.00153000
S	-6.53308800	-0.10589100	-0.01224900
H	6.86950300	-1.03368900	-0.11836900
H	-6.61935100	-1.44845000	0.05164000

HS-stilbene-SH twisted structure

C	0.44773000	0.33651000	-0.00648200
C	1.91044200	0.19836200	-0.00507700
C	2.70345000	1.35748700	0.00064900
C	2.57832200	-1.04054200	-0.00893700

C	3.96454500	-1.11484200	-0.00613300
C	-1.93780700	-0.40060500	0.00034300
C	-2.65301300	-0.29528400	1.20227100
C	-4.02905500	-0.08047700	1.20830900
C	-4.73012600	0.03191800	0.00103000
C	-4.02987600	-0.07354600	-1.20597200
C	-2.65251700	-0.28907400	-1.20010200
H	-4.55685400	0.00160500	2.15407200
H	-4.55313100	0.01401200	-2.15382700
H	-2.12227700	-0.36754500	-2.14477900
C	-0.46848800	-0.64451900	0.00014900
C	4.09425000	1.29575100	0.00347600
C	4.74007300	0.05497200	0.00029400
H	4.44981100	-2.08651600	-0.00916900
H	2.00873300	-1.96450100	-0.01478300
H	4.67113000	2.21593900	0.00816300
H	2.21919300	2.33061900	0.00349000
H	-2.12243000	-0.37872100	2.14624000
H	0.08668100	1.36466900	-0.01269200
H	-0.15107700	-1.68762900	0.00989400
S	6.51723700	-0.12038300	0.00329100
S	-6.49533300	0.30916900	0.08077600
H	6.80832300	1.19456300	0.01311700
H	-6.71613600	0.32626800	-1.24770400

Sample input file with the coordinates for the full system of Au_L-

S-Ph-amide-Ph-S-Au_R

```
# Sample input for the L-S-Ph-amide-Ph-S-R system
```

```
%block LatticeVectors
  30.000    0.000    0.00000
  0.000    30.000    0.00000
  0.00000    0.00000  45.74497
%endblock LatticeVectors
```

```
PAO.BasisType split
PAO.SplitNorm 0.15
%block PAO.BasisSizes
  Au  DZ
```



```

S    DZ
C    DZ
H    DZ
N    DZ
O    DZ
%endblock PAO.BasisSizes
PAO.EnergyShift 0.01 eV

%block kgrid_Monkhorst_Pack
  1    0    0    0.0
  0    1    0    0.0
  0    0    1    0.0
%endblock kgrid_Monkhorst_Pack
BandLinesScale          ReciprocalLatticeVectors

XC.functional           GGA
XC.authors              PBE
MeshCutoff 200.0 Ry

ElectronicTemperature    300 K
MaxSCFIterations        30000
DM.MixingWeight         0.02
DM.NumberPulay          4
DM.Tolerance            1.0D-5
DM.UseSaveDM            T

SolutionMethod          transiesta

TS.HSFileLeft AuLeads.TSHS
TS.NumUsedAtomsLeft 20
TS.HSFileRight AuLeads.TSHS
TS.NumUsedAtomsRight 20

TS.TBT.NPoints 1000
TS.TBT.Emin -1.0 eV
TS.TBT.Emax 1.0 eV
TS.TBT.OutputRegionData False
TS.BiasContour.NumPoints 100
TS.ComplexContour.Emin -20.0 Ry
TS.ComplexContour.NumCircle 100
TS.ComplexContour.NumLine 20
TS.ComplexContour.NumPoles 10
TS.BiasContour.Eta 10D-4 Ry
TS.Voltage VOLTAGE eV

```

```

%block ChemicalSpeciesLabel
  1          79 Au
  2          16 S
  3           6 C
  4           1 H
  5           7 N
  6           8 O
%endblock ChemicalSpeciesLabel

```

```
AtomicCoordinatesFormat  Ang
```

```

%block AtomicCoordinatesAndAtomicSpecies
-0.83280000  -1.44250000   0.00000000   1  Au   1
 1.66560000   0.00000000   0.00000000   1  Au   2
-0.83280000   1.44250000   0.00000000   1  Au   3
 0.00000000  -2.88500000   2.35560000   1  Au   4
-2.49850000  -1.44250000   2.35560000   1  Au   5
 2.49850000  -1.44250000   2.35560000   1  Au   6
 0.00000000   0.00000000   2.35560000   1  Au   7
-2.49850000   1.44250000   2.35560000   1  Au   8
 2.49850000   1.44250000   2.35560000   1  Au   9
 0.00000000   2.88500000   2.35560000   1  Au  10
-0.83280000  -1.44250000   4.71120000   1  Au  11
 1.66560000   0.00000000   4.71120000   1  Au  12
-0.83280000   1.44250000   4.71120000   1  Au  13
 0.00000000  -2.88500000   7.06680000   1  Au  14
-2.49850000  -1.44250000   7.06680000   1  Au  15
 2.49850000  -1.44250000   7.06680000   1  Au  16
 0.00000000   0.00000000   7.06680000   1  Au  17
-2.49850000   1.44250000   7.06680000   1  Au  18
 2.49850000   1.44250000   7.06680000   1  Au  19
 0.00000000   2.88500000   7.06680000   1  Au  20
-0.83280000  -1.44250000   9.42240000   1  Au  21
 1.66560000   0.00000000   9.42240000   1  Au  22
-0.83280000   1.44250000   9.42240000   1  Au  23
 0.00000000  -2.88500000  11.77800000   1  Au  24
-2.49850000  -1.44250000  11.77800000   1  Au  25
 2.49850000  -1.44250000  11.77800000   1  Au  26
 0.00000000   0.00000000  11.77800000   1  Au  27
-2.49850000   1.44250000  11.77800000   1  Au  28
 2.49850000   1.44250000  11.77800000   1  Au  29
 0.00000000   2.88500000  11.77800000   1  Au  30
 0.00000000   0.00000000  14.09800000   2  S   31
-0.02008000  -0.15150000  15.88056000   3  C   32
-0.42057000  -2.27107000  15.97585000   4  H   33
 0.38445000   1.95454000  16.15478000   4  H   34

```

-0.24329000	-1.36252000	16.54395000	3	C	35
0.21218000	1.00008000	16.64291000	3	C	36
-0.24178000	-1.43752000	17.93531000	3	C	37
0.21987000	0.93217000	18.03045000	3	C	38
-0.41059000	-2.37921000	18.43713000	4	H	39
0.40020000	1.83800000	18.60507000	4	H	40
-0.00712000	-0.28250000	18.69564000	3	C	41
0.00116000	-0.26285000	20.10544000	5	N	42
0.21745000	0.63350000	20.51516000	4	H	43
-0.22057000	-2.48930000	20.62852000	6	O	44
-0.11852000	-1.31878000	20.98318000	3	C	45
-0.79067000	1.10520000	22.26360000	4	H	46
-0.09830000	-0.94365000	22.43672000	3	C	47
-0.44771000	0.32071000	22.93260000	3	C	48
0.50301000	-2.92806000	22.96561000	4	H	49
0.26016000	-1.94355000	23.35031000	3	C	50
-0.41669000	0.58647000	24.29800000	3	C	51
-0.70402000	1.56816000	24.66218000	4	H	52
0.30380000	-1.68495000	24.71478000	3	C	53
-0.03158000	-0.41336000	25.20177000	3	C	54
0.59678000	-2.47416000	25.40064000	4	H	55
0.00000000	0.00000000	26.93577000	2	S	56
0.00000000	-2.88500000	29.25577000	1	Au	57
-2.49850000	-1.44250000	29.25577000	1	Au	58
2.49850000	-1.44250000	29.25577000	1	Au	59
0.00000000	0.00000000	29.25577000	1	Au	60
-2.49850000	1.44250000	29.25577000	1	Au	61
2.49850000	1.44250000	29.25577000	1	Au	62
0.00000000	2.88500000	29.25577000	1	Au	63
-0.83280000	-1.44250000	31.61137000	1	Au	64
1.66560000	0.00000000	31.61137000	1	Au	65
-0.83280000	1.44250000	31.61137000	1	Au	66
0.00000000	-2.88500000	33.96697000	1	Au	67
-2.49850000	-1.44250000	33.96697000	1	Au	68
2.49850000	-1.44250000	33.96697000	1	Au	69
0.00000000	0.00000000	33.96697000	1	Au	70
-2.49850000	1.44250000	33.96697000	1	Au	71
2.49850000	1.44250000	33.96697000	1	Au	72
0.00000000	2.88500000	33.96697000	1	Au	73
-0.83280000	-1.44250000	36.32257000	1	Au	74
1.66560000	0.00000000	36.32257000	1	Au	75
-0.83280000	1.44250000	36.32257000	1	Au	76
0.00000000	-2.88500000	38.67817000	1	Au	77
-2.49850000	-1.44250000	38.67817000	1	Au	78
2.49850000	-1.44250000	38.67817000	1	Au	79
0.00000000	0.00000000	38.67817000	1	Au	80

-2.49850000	1.44250000	38.67817000	1	Au	81
2.49850000	1.44250000	38.67817000	1	Au	82
0.00000000	2.88500000	38.67817000	1	Au	83
-0.83280000	-1.44250000	41.03377000	1	Au	84
1.66560000	0.00000000	41.03377000	1	Au	85
-0.83280000	1.44250000	41.03377000	1	Au	86
0.00000000	-2.88500000	43.38937000	1	Au	87
-2.49850000	-1.44250000	43.38937000	1	Au	88
2.49850000	-1.44250000	43.38937000	1	Au	89
0.00000000	0.00000000	43.38937000	1	Au	90
-2.49850000	1.44250000	43.38937000	1	Au	91
2.49850000	1.44250000	43.38937000	1	Au	92
0.00000000	2.88500000	43.38937000	1	Au	93

%endblock AtomicCoordinatesAndAtomicSpecies

References

- (1) Zahid, F.; Paulsson, M.; Datta, S. In *Advanced Semiconductor and Organic Nano-Techniques III: Physics and Technology of Molecular and Biotechnology Systems*; Morkoç, H., Ed.; Academic Press: San Diego, 2003; Chapter 1 - Electrical Conduction through Molecules, pp 1 – 41.
- (2) Economou, E. *Green's Functions in Quantum Physics*, 3rd ed.; Springer Series in Solid-State Sciences; Springer, 2006.