

Supplemental Material: Efficient multiphoton sampling of molecular vibronic spectra on a superconducting bosonic processor

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I. NUMERICAL FRANCK-CONDON DATA

Additional experimental data is provided in this section. TABLE I provides an overview of the different molecular processes that are simulated and corresponding information regarding post-selection, systematic offsets, and distance metrics.

The data for each molecular process in the following tables is calculated as follows. For the “single-bit extraction” scheme, the probability and standard error for a given joint photon number of interest is:

$$q_{n',m'}^{\text{meas}} = \frac{n_{n',m'}^{ee}}{N_{n',m'}^{\text{runs}}} \quad (\text{S33})$$

$$\sigma_{n',m'} = \sqrt{\frac{q_{n',m'}^{\text{meas}}(1 - q_{n',m'}^{\text{meas}})}{N_{n',m'}^{\text{runs}}}} \quad (\text{S34})$$

where $n_{n',m'}^{ee}$ is the number of counts where both ancillas are measured in their excited state, indicating a measure for population in $|n', m'\rangle$ (see Eq. 4 in the main text), and $N_{n',m'}^{\text{runs}}$ is the total number of runs of the experiment for probing $|n', m'\rangle$. The number of runs varies slightly among different final states due to varying post-selection probabilities. The correction protocol outlined in Appendix E of the main text is then applied to $q_{n',m'}^{\text{meas}}$ to retrieve a new probability distribution $p_{n',m'}^{\text{meas}}$. The standard error $\sigma_{n',m'}$ is truncated to one significant digit and $p_{n',m'}^{\text{meas}}$ is then rounded to the precision set by $\sigma_{n',m'}$. The data reported is $p_{n',m'}^{\text{meas}} \pm \sigma_{n',m'}$ only for probabilities with significant support relative to the precision of the experiment ($p_{n',m'}^{\text{ideal}} \gtrsim 10^{-4}$).

The same method (sans the correction protocol) is applied to the data for the “sampling” scheme, except there the probabilities and standard error are given by:

$$q_{n',m'}^{\text{meas}} = \frac{n_{n',m'}}{N_{\text{runs}}} \quad (\text{S35})$$

$$\sigma_{n',m'} = \sqrt{\frac{q_{n',m'}^{\text{meas}}(1 - q_{n',m'}^{\text{meas}})}{N_{\text{runs}}}} \quad (\text{S36})$$

where $n_{n',m'}$ is the number of times the joint photon number $|n', m'\rangle$ is sampled from the total number of runs of the experiment N_{runs} .

Sometimes, there will be no counts reported for a given $|n', m'\rangle$ (i.e., $n_{n',m'}^{ee}$ or $n_{n',m'} = 0$). In this case, we simply report a probability of zero. Furthermore, sometimes the correction protocol will return negative elements in the probability distribution due to statistical noise; these unphysical cases are also nulled and a zero is reported. All distances $D = \frac{1}{2} \sum_{i=0}^{n_{\text{max}}} \sum_{j=0}^{n_{\text{max}}} |p_{ij}^{\text{meas}} - p_{ij}^{\text{ideal}}|$ are calculated after this correction process, with the corresponding values for n_{max} specified in TABLE I.

Full time-domain master equation simulations are performed using QuTiP and consider only the Hilbert space of the two cavities with $n_{\text{max}} = 30$. Each Gaussian operation is simulated by evolving the associated Hamiltonian term, while also including the corresponding self-Kerr terms and photon loss for each operation. While squeezing cavity A, for instance, the native self-Kerr and photon loss rates for cavity B are used, assuming that the pumped process for squeezing cavity A does not change the participation of cavity B in any nonlinear lossy modes (and vice-versa). The simulation also takes into account photon loss during the verification measurement, which takes $2.5 \mu\text{s}$. The simulation does not consider imperfect state preparation and systematic errors in calibrations, which we believe to account for the remaining difference between the measured distances for the “single-bit extraction” scheme and predicted distances from the master equation simulations.

Molecular transition	Initial state (n, m)	Percentage of data kept	n_{\max}	t_A, t_B f_A, f_B	Distance to ideal distribution			Figure
					Single-bit extraction	Sampling	Master Equation	
$\text{H}_2\text{O} \xrightarrow{h\nu} \text{H}_2\text{O}^+(\tilde{B}^2B_2) + e^-$	(0, 0)	$\sim 95\%$	16	0.937, 0.946 0.005, 0.002	0.049(1)	0.151(9)	0.0123	FIG. 7
$\text{O}_3^- \xrightarrow{h\nu} \text{O}_3 + e^-$	(0, 0)	$\sim 96\%$	12	0.937, 0.948 0.005, 0.002	0.039(9)	0.075(2)	0.0052	FIG. 8
	(1, 0)	$\sim 95\%$	10	0.937, 0.950 0.004, 0.002	0.057(5)	0.085(5)	0.0131	FIG. 9
	(1, 2)	$\sim 93\%$	12	0.938, 0.950 0.004, 0.001	0.105(3)	0.148(4)	0.0217	FIG. 10
$\text{NO}_2^- \xrightarrow{h\nu} \text{NO}_2 + e^-$	(0, 0)	$\sim 94\%$	12	0.935, 0.943 0.005, 0.003	0.034(0)	0.110(9)	0.0331	FIG. 11
	(1, 0)	$\sim 92\%$	14	0.934, 0.951 0.004, 0.002	0.202(2)	0.209(7)	0.1269	FIG. 12
$\text{SO}_2 \xrightarrow{h\nu} \text{SO}_2^+ + e^-$	(0, 0)	$\sim 96\%$	12	0.938, 0.950 0.004, 0.002	0.019(6)	0.095(3)	0.0065	FIG. 13
	(0, 1)	$\sim 94\%$	12	0.931, 0.951 0.004, 0.001	0.063(7)	0.136(6)	0.0213	FIG. 14

TABLE I. **Summary of experimental data.** List of molecular processes simulated and corresponding post-selection probabilities, maximum photon number probed with the “single-bit extraction” measurement scheme, and distances. Transmon offsets are independently measured after each dataset is taken.

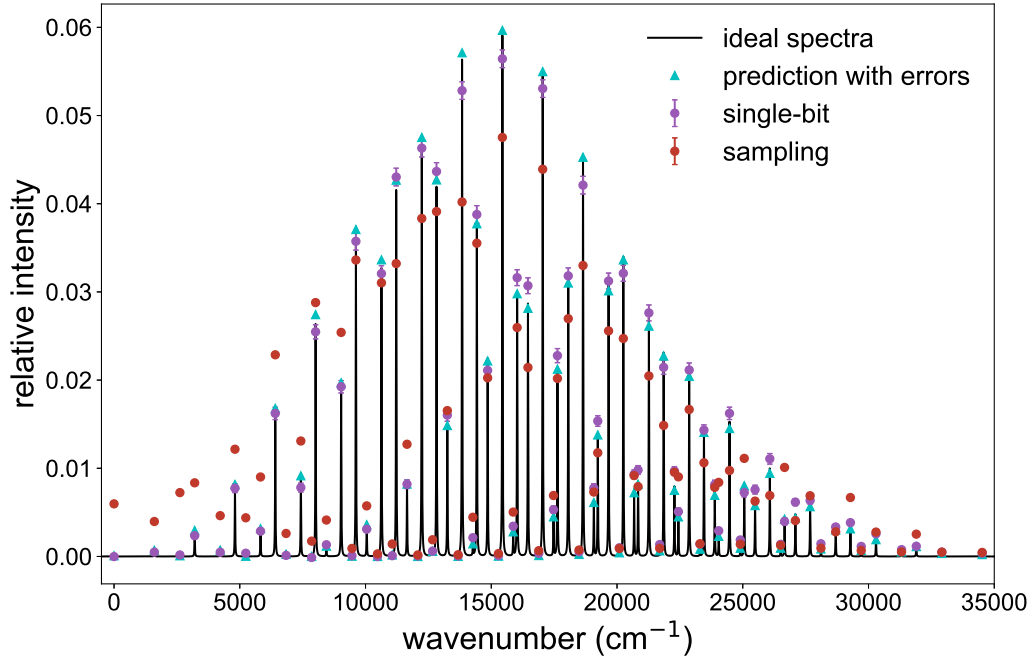


FIG. 1. Photoionization of neutral water to the (\tilde{B}^2B_2) excited state of the cation starting in the vibrationless state $n = 0$, $m = 0$.

$\text{H}_2\text{O} \xrightarrow{h\nu} \text{H}_2\text{O}^+(\tilde{B}^2B_2) + e^-$ starting in ($n = 0, m = 0$)				
(n', m')	Classically calculated	Master equation simulation	Single-bit extraction	Sampling
(0,0)	7.92E-05	8.26E-05	0	0.006 ± 0.0001
(0,1)	6.67E-04	7.01E-04	0.0005 ± 0.0002	0.00396 ± 0.0001
(0,2)	2.80E-03	2.94E-03	0.0024 ± 0.0003	0.0084 ± 0.0001
(0,3)	7.76E-03	8.15E-03	0.0077 ± 0.0005	0.0122 ± 0.0002
(0,4)	1.60E-02	1.68E-02	0.0162 ± 0.0007	0.0229 ± 0.0002
(0,5)	2.64E-02	2.74E-02	0.0255 ± 0.0008	0.0288 ± 0.0003
(0,6)	3.59E-02	3.70E-02	0.0357 ± 0.001	0.0336 ± 0.0003
(0,7)	4.16E-02	4.26E-02	0.043 ± 0.001	0.0332 ± 0.0003
(0,8)	4.19E-02	4.27E-02	0.044 ± 0.001	0.0391 ± 0.0003
(0,9)	3.73E-02	3.77E-02	0.039 ± 0.001	0.0355 ± 0.0003
(0,10)	2.96E-02	2.98E-02	0.0316 ± 0.0009	0.026 ± 0.0003
(0,11)	2.13E-02	2.12E-02	0.0228 ± 0.0008	0.0202 ± 0.0002
(0,12)	1.39E-02	1.38E-02	0.0154 ± 0.0006	0.0118 ± 0.0002
(0,13)	8.33E-03	8.20E-03	0.0098 ± 0.0005	0.0079 ± 0.0001
(0,14)	4.60E-03	4.50E-03	0.0051 ± 0.0004	0.009 ± 0.0001
(0,15)	2.36E-03	2.29E-03	0.0029 ± 0.0003	0.0084 ± 0.0001
(1,0)	7.82E-05	8.11E-05	0.0002 ± 0.0002	0.0072 ± 0.0001
(1,1)	6.91E-04	7.19E-04	0.0005 ± 0.0002	0.0046 ± 0.0001
(1,2)	3.03E-03	3.16E-03	0.0029 ± 0.0003	0.009 ± 0.0001
(1,3)	8.81E-03	9.14E-03	0.0078 ± 0.0005	0.0131 ± 0.0002
(1,4)	1.91E-02	1.97E-02	0.0192 ± 0.0007	0.0254 ± 0.0002
(1,5)	3.28E-02	3.36E-02	0.0321 ± 0.0009	0.031 ± 0.0003
(1,6)	4.66E-02	4.75E-02	0.046 ± 0.001	0.0383 ± 0.0003
(1,7)	5.63E-02	5.71E-02	0.053 ± 0.001	0.0402 ± 0.0003
(1,8)	5.92E-02	5.97E-02	0.056 ± 0.001	0.0475 ± 0.0003
(1,9)	5.49E-02	5.50E-02	0.053 ± 0.001	0.0439 ± 0.0003

(1,10)	4.55E-02	4.53E-02	0.042 ± 0.001	0.033 ± 0.0003
(1,11)	3.40E-02	3.37E-02	0.0321 ± 0.0009	0.0247 ± 0.0002
(1,12)	2.32E-02	2.28E-02	0.0214 ± 0.0008	0.0149 ± 0.0002
(1,13)	1.44E-02	1.41E-02	0.0143 ± 0.0006	0.0106 ± 0.0002
(1,14)	8.30E-03	8.04E-03	0.0073 ± 0.0005	0.0111 ± 0.0002
(1,15)	4.42E-03	4.25E-03	0.004 ± 0.0004	0.0101 ± 0.0002
(2,0)	2.61E-05	2.68E-05	0.0004 ± 0.0001	0.0044 ± 0.0001
(2,1)	2.47E-04	2.52E-04	0.0001 ± 0.0001	0.00261 ± 8E-05
(2,2)	1.15E-03	1.17E-03	0.0013 ± 0.0002	0.0041 ± 0.0001
(2,3)	3.57E-03	3.60E-03	0.0031 ± 0.0003	0.0057 ± 0.0001
(2,4)	8.19E-03	8.21E-03	0.0082 ± 0.0005	0.0127 ± 0.0002
(2,5)	1.49E-02	1.48E-02	0.016 ± 0.0007	0.0165 ± 0.0002
(2,6)	2.25E-02	2.22E-02	0.0211 ± 0.0008	0.0203 ± 0.0002
(2,7)	2.87E-02	2.81E-02	0.0307 ± 0.0009	0.0214 ± 0.0002
(2,8)	3.19E-02	3.10E-02	0.0318 ± 0.0009	0.027 ± 0.0003
(2,9)	3.12E-02	3.01E-02	0.0312 ± 0.0009	0.0256 ± 0.0002
(2,10)	2.72E-02	2.61E-02	0.0276 ± 0.0009	0.0205 ± 0.0002
(2,11)	2.14E-02	2.04E-02	0.0211 ± 0.0008	0.0167 ± 0.0002
(2,12)	1.53E-02	1.45E-02	0.0162 ± 0.0007	0.0097 ± 0.0002
(2,13)	1.00E-02	9.46E-03	0.0111 ± 0.0006	0.0069 ± 0.0001
(2,14)	6.02E-03	5.67E-03	0.0064 ± 0.0004	0.0069 ± 0.0001
(2,15)	3.36E-03	3.14E-03	0.0038 ± 0.0003	0.0067 ± 0.0001
(3,0)	2.78E-06	3.35E-06	0	0.00174 ± 7E-05
(3,1)	2.97E-05	3.43E-05	8E-05 ± 8E-05	0.00091 ± 5E-05
(3,2)	1.56E-04	1.74E-04	9E-05 ± 8E-05	0.00142 ± 6E-05
(3,3)	5.37E-04	5.79E-04	0.0006 ± 0.0002	0.00189 ± 7E-05
(3,4)	1.36E-03	1.43E-03	0.0021 ± 0.0003	0.0044 ± 0.0001
(3,5)	2.73E-03	2.79E-03	0.0034 ± 0.0003	0.005 ± 0.0001
(3,6)	4.50E-03	4.49E-03	0.0053 ± 0.0004	0.0069 ± 0.0001
(3,7)	6.26E-03	6.12E-03	0.0078 ± 0.0005	0.0073 ± 0.0001
(3,8)	7.52E-03	7.22E-03	0.0093 ± 0.0005	0.0092 ± 0.0002
(3,9)	7.94E-03	7.50E-03	0.0097 ± 0.0005	0.0095 ± 0.0002
(3,10)	7.46E-03	6.94E-03	0.0082 ± 0.0005	0.0078 ± 0.0001
(3,11)	6.29E-03	5.78E-03	0.0076 ± 0.0005	0.0063 ± 0.0001
(3,12)	4.81E-03	4.37E-03	0.0062 ± 0.0004	0.0041 ± 0.0001
(3,13)	3.36E-03	3.01E-03	0.0033 ± 0.0003	0.00279 ± 8E-05
(3,14)	2.16E-03	1.91E-03	0.0026 ± 0.0003	0.00277 ± 8E-05
(3,15)	1.28E-03	1.12E-03	0.0012 ± 0.0002	0.00253 ± 8E-05
(4,0)	2.66E-09	1.53E-07	0	0.00029 ± 3E-05
(4,1)	1.45E-07	1.74E-06	6E-05 ± 5E-05	0.00017 ± 2E-05
(4,2)	1.73E-06	9.99E-06	6E-05 ± 6E-05	0.0002 ± 2E-05
(4,3)	1.02E-05	3.79E-05	0.0001 ± 8E-05	0.00033 ± 3E-05
(4,4)	3.86E-05	1.06E-04	0.0004 ± 0.0001	0.00065 ± 4E-05
(4,5)	1.06E-04	2.33E-04	0.0005 ± 0.0002	0.00073 ± 4E-05
(4,6)	2.25E-04	4.19E-04	0.001 ± 0.0002	0.0009 ± 5E-05
(4,7)	3.88E-04	6.34E-04	0.0014 ± 0.0002	0.00092 ± 5E-05
(4,8)	5.63E-04	8.24E-04	0.0014 ± 0.0002	0.00144 ± 6E-05
(4,9)	7.01E-04	9.36E-04	0.0019 ± 0.0003	0.00139 ± 6E-05
(4,10)	7.63E-04	9.43E-04	0.0014 ± 0.0002	0.00127 ± 6E-05
(4,11)	7.36E-04	8.50E-04	0.0014 ± 0.0002	0.00094 ± 5E-05
(4,12)	6.36E-04	6.92E-04	0.0011 ± 0.0002	0.00067 ± 4E-05
(4,13)	4.97E-04	5.13E-04	0.0008 ± 0.0002	0.00051 ± 4E-05
(4,14)	3.54E-04	3.49E-04	0.0005 ± 0.0001	0.00049 ± 3E-05
(4,15)	2.31E-04	2.18E-04	0.0004 ± 0.0001	0.00046 ± 3E-05

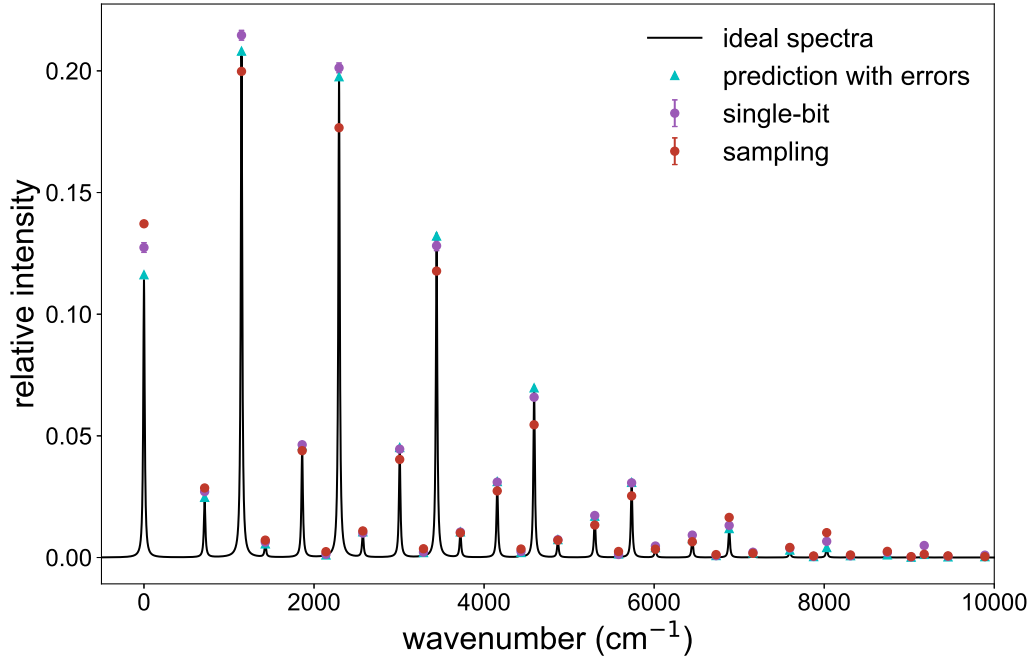


FIG. 2. Photoionization of the ozone anion to neutral ozone starting in the vibrationless state $n = 0$, $m = 0$.

(n', m')	$\text{O}_3^- \xrightarrow{h\nu} \text{O}_3 + e^-$ starting in $(n = 0, m = 0)$			
	Classically calculated	Master equation simulation	Single-bit extraction	Sampling
(0,0)	1.14E-01	1.16E-01	0.127 ± 0.002	0.1372 ± 0.0005
(0,1)	2.42E-02	2.47E-02	0.0271 ± 0.0008	0.0286 ± 0.0003
(0,2)	5.57E-03	5.60E-03	0.0063 ± 0.0004	0.0071 ± 0.0001
(0,3)	1.06E-03	1.06E-03	0.0013 ± 0.0002	$0.0024 \pm 8\text{E-}05$
(1,0)	2.06E-01	2.08E-01	0.215 ± 0.002	0.1998 ± 0.0006
(1,1)	4.55E-02	4.60E-02	0.046 ± 0.001	0.0439 ± 0.0003
(1,2)	1.06E-02	1.06E-02	0.01 ± 0.0005	0.0109 ± 0.0002
(1,3)	2.06E-03	2.04E-03	0.0022 ± 0.0003	0.0036 ± 0.0001
(2,0)	1.98E-01	1.98E-01	0.201 ± 0.002	0.1766 ± 0.0006
(2,1)	4.52E-02	4.52E-02	0.045 ± 0.001	0.0403 ± 0.0003
(2,2)	1.07E-02	1.05E-02	0.0104 ± 0.0005	0.0101 ± 0.0002
(2,3)	2.11E-03	2.06E-03	0.0025 ± 0.0003	$0.00349 \pm 9\text{E-}05$
(3,0)	1.33E-01	1.32E-01	0.128 ± 0.002	0.1177 ± 0.0005
(3,1)	3.15E-02	3.12E-02	0.031 ± 0.0009	0.0274 ± 0.0003
(3,2)	7.54E-03	7.37E-03	0.0073 ± 0.0005	0.0071 ± 0.0001
(3,3)	1.51E-03	1.47E-03	0.0012 ± 0.0002	$0.00248 \pm 8\text{E-}05$
(4,0)	7.08E-02	6.97E-02	0.066 ± 0.001	0.0546 ± 0.0004
(4,1)	1.73E-02	1.70E-02	0.0173 ± 0.0007	0.0133 ± 0.0002
(4,2)	4.19E-03	4.06E-03	0.0047 ± 0.0004	$0.00342 \pm 9\text{E-}05$
(4,3)	8.51E-04	8.19E-04	0.0008 ± 0.0002	$0.0012 \pm 6\text{E-}05$
(5,0)	3.15E-02	3.09E-02	0.0307 ± 0.0009	0.0253 ± 0.0003
(5,1)	7.91E-03	7.74E-03	0.0092 ± 0.0005	0.0065 ± 0.0001
(5,2)	1.94E-03	1.87E-03	0.0022 ± 0.0003	$0.00173 \pm 7\text{E-}05$
(5,3)	4.01E-04	3.83E-04	0.0004 ± 0.0001	$0.00064 \pm 4\text{E-}05$
(6,0)	1.22E-02	1.19E-02	0.0132 ± 0.0007	0.0165 ± 0.0002
(6,1)	3.15E-03	3.07E-03	0.0039 ± 0.0004	0.0041 ± 0.0001
(6,2)	7.83E-04	7.52E-04	0.0007 ± 0.0002	$0.00105 \pm 5\text{E-}05$

(6,3)	1.64E-04	1.56E-04	$0.00024 \pm 9E-05$	$0.00036 \pm 3E-05$
(7,0)	4.23E-03	4.10E-03	0.0066 ± 0.0005	0.0102 ± 0.0002
(7,1)	1.12E-03	1.09E-03	0.0021 ± 0.0003	$0.00252 \pm 8E-05$
(7,2)	2.81E-04	2.69E-04	0.0005 ± 0.0001	$0.00068 \pm 4E-05$
(7,3)	5.96E-05	5.64E-05	$0.00019 \pm 8E-05$	$0.00017 \pm 2E-05$
(8,0)	1.33E-03	1.28E-03	0.0049 ± 0.0005	$0.00132 \pm 6E-05$
(8,1)	3.61E-04	3.48E-04	0.001 ± 0.0002	$0.00035 \pm 3E-05$
(8,2)	9.18E-05	8.73E-05	0.0002 ± 0.0001	$9E-05 \pm 2E-05$
(8,3)	1.97E-05	1.85E-05	$0.00013 \pm 7E-05$	$2.8E-05 \pm 8E-06$
(9,0)	3.86E-04	3.68E-04	0.0047 ± 0.0005	$0.00156 \pm 6E-05$
(9,1)	1.07E-04	1.03E-04	0.0012 ± 0.0002	$0.00038 \pm 3E-05$
(9,2)	2.76E-05	2.60E-05	0.0004 ± 0.0001	$8E-05 \pm 1E-05$
(9,3)	5.98E-06	5.59E-06	$0.00013 \pm 7E-05$	$3.6E-05 \pm 1E-05$

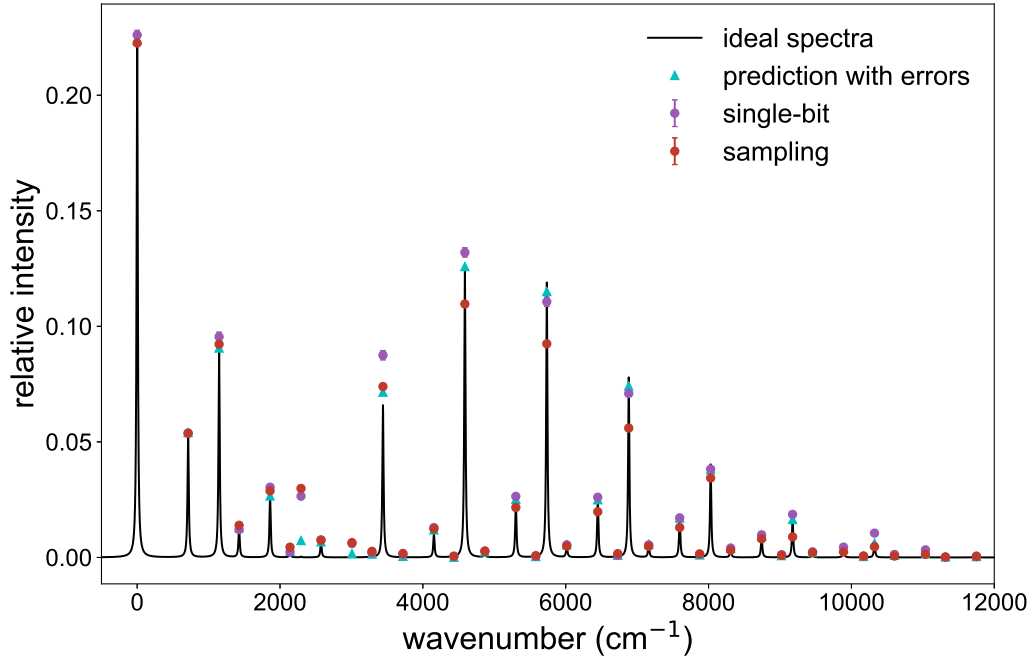


FIG. 3. Photoionization of the ozone anion to neutral ozone starting with one quanta in the symmetric-stretching mode and zero in the bending mode $n = 1$, $m = 0$.

(n', m')	$O_3^- \xrightarrow{h\nu} O_3 + e^-$ starting in $(n = 1, m = 0)$			
	Classically calculated	Master equation simulation	Single-bit extraction	Sampling
(0,0)	2.24E-01	2.23E-01	0.226 ± 0.002	0.2226 ± 0.0007
(0,1)	5.42E-02	5.40E-02	0.054 ± 0.001	0.0537 ± 0.0004
(0,2)	1.31E-02	1.28E-02	0.0121 ± 0.0006	0.0139 ± 0.0002
(0,3)	2.64E-03	2.56E-03	0.0023 ± 0.0003	0.0044 ± 0.0001
(1,0)	8.96E-02	9.09E-02	0.095 ± 0.002	0.0923 ± 0.0005
(1,1)	2.67E-02	2.69E-02	0.0304 ± 0.0009	0.0288 ± 0.0003
(1,2)	6.90E-03	6.83E-03	0.0075 ± 0.0005	0.0075 ± 0.0001
(1,3)	1.51E-03	1.48E-03	0.002 ± 0.0003	$0.00263 \pm 9E-05$
(2,0)	3.14E-04	5.76E-03	0.0265 ± 0.0009	0.0299 ± 0.0003
(2,1)	2.69E-04	1.39E-03	0.006 ± 0.0004	0.0064 ± 0.0001
(2,2)	1.59E-04	4.07E-04	0.0013 ± 0.0002	$0.00174 \pm 7E-05$
(2,3)	6.61E-05	1.11E-04	0.0004 ± 0.0001	$0.00059 \pm 4E-05$
(3,0)	6.59E-02	6.96E-02	0.088 ± 0.002	0.074 ± 0.0004
(3,1)	1.05E-02	1.14E-02	0.0129 ± 0.0006	0.0124 ± 0.0002
(3,2)	2.11E-03	2.27E-03	0.0028 ± 0.0003	$0.00267 \pm 9E-05$
(3,3)	3.34E-04	3.67E-04	0.0005 ± 0.0001	$0.00078 \pm 5E-05$
(4,0)	1.26E-01	1.25E-01	0.132 ± 0.002	0.1097 ± 0.0005
(4,1)	2.51E-02	2.49E-02	0.0264 ± 0.0009	0.0216 ± 0.0002
(4,2)	5.55E-03	5.42E-03	0.0054 ± 0.0004	0.0049 ± 0.0001
(4,3)	1.01E-03	9.81E-04	0.0009 ± 0.0002	$0.00165 \pm 7E-05$
(5,0)	1.19E-01	1.16E-01	0.111 ± 0.002	0.0925 ± 0.0005
(5,1)	2.58E-02	2.51E-02	0.0261 ± 0.0009	0.0198 ± 0.0002
(5,2)	5.94E-03	5.68E-03	0.0054 ± 0.0004	0.0048 ± 0.0001
(5,3)	1.14E-03	1.08E-03	0.0014 ± 0.0002	$0.00153 \pm 7E-05$
(6,0)	7.79E-02	7.54E-02	0.071 ± 0.001	0.056 ± 0.0004
(6,1)	1.79E-02	1.73E-02	0.0171 ± 0.0007	0.013 ± 0.0002

(6,2)	4.23E-03	4.01E-03	0.0041 ± 0.0004	0.00313 ± 9E-05
(6,3)	8.33E-04	7.82E-04	0.0012 ± 0.0002	0.00091 ± 5E-05
(7,0)	4.03E-02	3.89E-02	0.038 ± 0.001	0.0344 ± 0.0003
(7,1)	9.69E-03	9.34E-03	0.0098 ± 0.0005	0.008 ± 0.0001
(7,2)	2.33E-03	2.21E-03	0.0024 ± 0.0003	0.00207 ± 8E-05
(7,3)	4.70E-04	4.40E-04	0.0004 ± 0.0001	0.00065 ± 4E-05
(8,0)	1.75E-02	1.69E-02	0.0186 ± 0.0008	0.0089 ± 0.0002
(8,1)	4.38E-03	4.22E-03	0.0045 ± 0.0004	0.00239 ± 8E-05
(8,2)	1.07E-03	1.02E-03	0.0013 ± 0.0002	0.00059 ± 4E-05
(8,3)	2.20E-04	2.06E-04	0.00016 ± 8E-05	0.0002 ± 2E-05
(9,0)	6.66E-03	6.43E-03	0.0105 ± 0.0006	0.0046 ± 0.0001
(9,1)	1.72E-03	1.66E-03	0.0033 ± 0.0003	0.00132 ± 6E-05
(9,2)	4.28E-04	4.05E-04	0.0006 ± 0.0002	0.00034 ± 3E-05
(9,3)	8.93E-05	8.37E-05	3E-05 ± 6E-05	0.00013 ± 2E-05

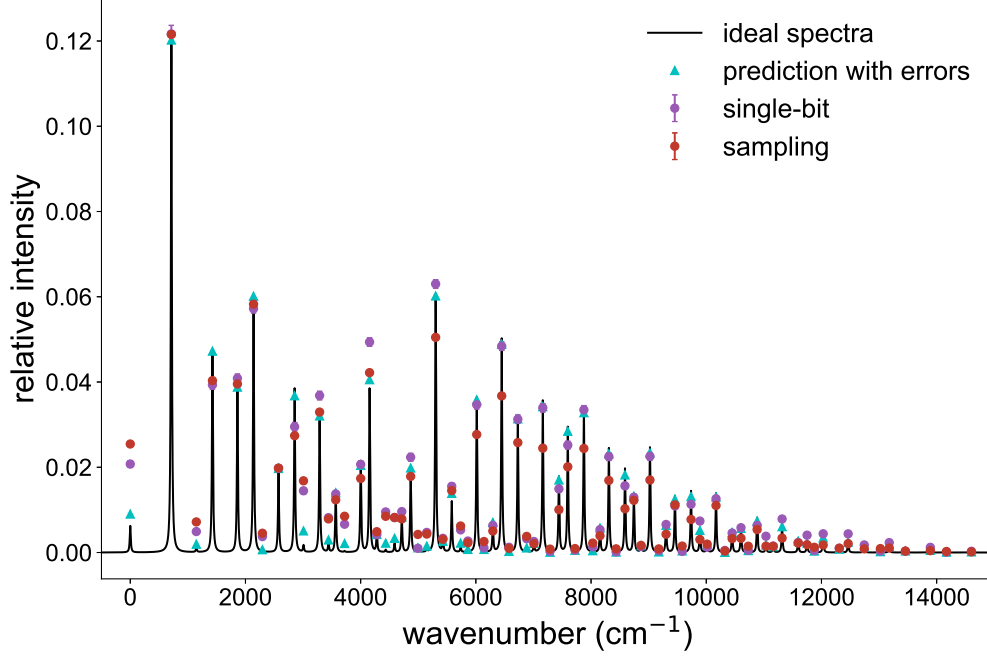


FIG. 4. Photoionization of the ozone anion to neutral ozone starting with one quanta in the symmetric-stretching mode and two in the bending mode $n = 1$, $m = 2$.

$O_3^- \xrightarrow{h\nu} O_3 + e^-$ starting in $(n = 1, m = 2)$				
(n', m')	Classically calculated	Master equation simulation	Single-bit extraction	Sampling
(0,0)	6.24E-03	9.10E-03	0.0208 ± 0.0008	0.0255 ± 0.0003
(0,1)	1.21E-01	1.20E-01	0.122 ± 0.002	0.1215 ± 0.0006
(0,2)	4.73E-02	4.75E-02	0.039 ± 0.001	0.0403 ± 0.0004
(0,3)	6.01E-02	6.02E-02	0.057 ± 0.001	0.0583 ± 0.0004
(0,4)	3.85E-02	3.65E-02	0.0295 ± 0.001	0.0274 ± 0.0003
(0,5)	1.46E-02	1.39E-02	0.0137 ± 0.0007	0.0124 ± 0.0002
(0,6)	4.57E-03	4.18E-03	0.0042 ± 0.0004	0.0049 ± 0.0001
(0,7)	1.21E-03	1.10E-03	0.001 ± 0.0002	0.0042 ± 0.0001
(1,0)	9.99E-04	1.99E-03	0.0049 ± 0.0004	0.0072 ± 0.0002
(1,1)	3.79E-02	3.89E-02	0.041 ± 0.001	0.0395 ± 0.0004
(1,2)	1.90E-02	1.99E-02	0.0198 ± 0.0008	0.0198 ± 0.0003
(1,3)	3.22E-02	3.22E-02	0.037 ± 0.001	0.0329 ± 0.0003
(1,4)	2.14E-02	2.04E-02	0.0207 ± 0.0008	0.0174 ± 0.0002
(1,5)	8.78E-03	8.31E-03	0.0096 ± 0.0006	0.0079 ± 0.0002
(1,6)	2.87E-03	2.63E-03	0.0033 ± 0.0003	0.0031 ± 0.0001
(1,7)	7.99E-04	7.23E-04	0.0011 ± 0.0002	0.00258 ± 0.0001
(2,0)	4.14E-04	6.10E-04	0.0037 ± 0.0004	0.0045 ± 0.0001
(2,1)	1.56E-03	4.26E-03	0.0145 ± 0.0007	0.0168 ± 0.0002
(2,2)	2.44E-04	1.72E-03	0.0066 ± 0.0005	0.0085 ± 0.0002
(2,3)	5.64E-04	1.90E-03	0.0095 ± 0.0006	0.0085 ± 0.0002
(2,4)	6.09E-04	1.33E-03	0.0047 ± 0.0004	0.0043 ± 0.0001
(2,5)	4.53E-04	6.66E-04	0.0027 ± 0.0003	$0.00229 \pm 9E-05$
(2,6)	1.95E-04	2.51E-04	0.0012 ± 0.0002	$0.00098 \pm 6E-05$
(2,7)	6.97E-05	7.85E-05	0.0003 ± 0.0001	$0.0008 \pm 5E-05$
(3,0)	2.02E-03	2.95E-03	0.0082 ± 0.0005	0.0079 ± 0.0002
(3,1)	3.84E-02	3.97E-02	0.049 ± 0.001	0.0422 ± 0.0004

(3,2)	1.87E-02	1.94E-02	0.0224 ± 0.0009	0.0179 ± 0.0003
(3,3)	1.20E-02	1.34E-02	0.0155 ± 0.0007	0.0145 ± 0.0002
(3,4)	6.36E-03	6.72E-03	0.0064 ± 0.0005	0.0051 ± 0.0001
(3,5)	1.83E-03	2.05E-03	0.0026 ± 0.0003	0.00206 ± 9E-05
(3,6)	4.70E-04	5.06E-04	0.0006 ± 0.0002	0.00093 ± 6E-05
(3,7)	9.93E-05	1.14E-04	0.0002 ± 0.0001	0.00086 ± 6E-05
(4,0)	2.05E-03	3.41E-03	0.0082 ± 0.0005	0.0082 ± 0.0002
(4,1)	6.06E-02	6.01E-02	0.063 ± 0.001	0.0505 ± 0.0004
(4,2)	3.64E-02	3.58E-02	0.035 ± 0.001	0.0277 ± 0.0003
(4,3)	3.11E-02	3.13E-02	0.031 ± 0.001	0.0258 ± 0.0003
(4,4)	1.77E-02	1.68E-02	0.0149 ± 0.0007	0.01 ± 0.0002
(4,5)	5.91E-03	5.69E-03	0.0054 ± 0.0004	0.0039 ± 0.0001
(4,6)	1.69E-03	1.55E-03	0.0014 ± 0.0002	0.00167 ± 8E-05
(4,7)	4.09E-04	3.80E-04	0.0003 ± 0.0001	0.00151 ± 7E-05
(5,0)	1.18E-03	2.29E-03	0.0053 ± 0.0004	0.0062 ± 0.0002
(5,1)	5.02E-02	4.93E-02	0.048 ± 0.001	0.0367 ± 0.0004
(5,2)	3.57E-02	3.46E-02	0.034 ± 0.001	0.0245 ± 0.0003
(5,3)	3.39E-02	3.31E-02	0.033 ± 0.001	0.0244 ± 0.0003
(5,4)	1.96E-02	1.82E-02	0.0156 ± 0.0007	0.0103 ± 0.0002
(5,5)	6.87E-03	6.40E-03	0.0066 ± 0.0005	0.0043 ± 0.0001
(5,6)	2.03E-03	1.81E-03	0.0012 ± 0.0002	0.00196 ± 8E-05
(5,7)	5.10E-04	4.57E-04	0.0005 ± 0.0002	0.00148 ± 7E-05
(6,0)	4.73E-04	1.13E-03	0.0032 ± 0.0003	0.0038 ± 0.0001
(6,1)	2.94E-02	2.89E-02	0.0252 ± 0.0009	0.0201 ± 0.0003
(6,2)	2.45E-02	2.36E-02	0.0225 ± 0.0009	0.0169 ± 0.0002
(6,3)	2.47E-02	2.37E-02	0.0226 ± 0.0009	0.017 ± 0.0002
(6,4)	1.44E-02	1.32E-02	0.0113 ± 0.0006	0.0078 ± 0.0002
(6,5)	5.18E-03	4.76E-03	0.0046 ± 0.0004	0.0033 ± 0.0001
(6,6)	1.56E-03	1.38E-03	0.0015 ± 0.0002	0.00143 ± 7E-05
(6,7)	4.00E-04	3.53E-04	0.0005 ± 0.0001	0.00122 ± 7E-05
(7,0)	1.44E-04	4.53E-04	0.0013 ± 0.0002	0.00221 ± 9E-05
(7,1)	1.37E-02	1.35E-02	0.013 ± 0.0007	0.0122 ± 0.0002
(7,2)	1.33E-02	1.28E-02	0.0109 ± 0.0006	0.0112 ± 0.0002
(7,3)	1.39E-02	1.33E-02	0.0126 ± 0.0007	0.011 ± 0.0002
(7,4)	8.18E-03	7.46E-03	0.0064 ± 0.0005	0.0054 ± 0.0001
(7,5)	3.00E-03	2.73E-03	0.0024 ± 0.0003	0.00221 ± 9E-05
(7,6)	9.14E-04	8.02E-04	0.001 ± 0.0002	0.00102 ± 6E-05
(7,7)	2.38E-04	2.08E-04	0.0004 ± 0.0001	0.00092 ± 6E-05
(8,0)	3.37E-05	1.55E-04	0.0005 ± 0.0002	0.00085 ± 6E-05
(8,1)	5.37E-03	5.35E-03	0.0074 ± 0.0005	0.003 ± 0.0001
(8,2)	6.07E-03	5.83E-03	0.0058 ± 0.0005	0.0033 ± 0.0001
(8,3)	6.54E-03	6.22E-03	0.0079 ± 0.0005	0.0034 ± 0.0001
(8,4)	3.86E-03	3.52E-03	0.0044 ± 0.0004	0.00176 ± 8E-05
(8,5)	1.44E-03	1.30E-03	0.0018 ± 0.0003	0.00085 ± 6E-05
(8,6)	4.42E-04	3.88E-04	0.0003 ± 0.0001	0.00033 ± 3E-05
(8,7)	1.17E-04	1.02E-04	0.0001 ± 7E-05	0.00023 ± 3E-05
(9,0)	5.90E-06	4.75E-05	0.0005 ± 0.0002	0.00035 ± 4E-05
(9,1)	1.84E-03	1.84E-03	0.0038 ± 0.0004	0.00149 ± 7E-05
(9,2)	2.42E-03	2.32E-03	0.0041 ± 0.0004	0.00184 ± 8E-05
(9,3)	2.67E-03	2.53E-03	0.0044 ± 0.0004	0.00208 ± 9E-05
(9,4)	1.58E-03	1.44E-03	0.0023 ± 0.0003	0.00105 ± 6E-05
(9,5)	5.95E-04	5.39E-04	0.0012 ± 0.0002	0.00044 ± 4E-05
(9,6)	1.85E-04	1.62E-04	0.0003 ± 0.0001	0.00016 ± 2E-05
(9,7)	4.94E-05	4.28E-05	0.00012 ± 7E-05	0.00015 ± 2E-05

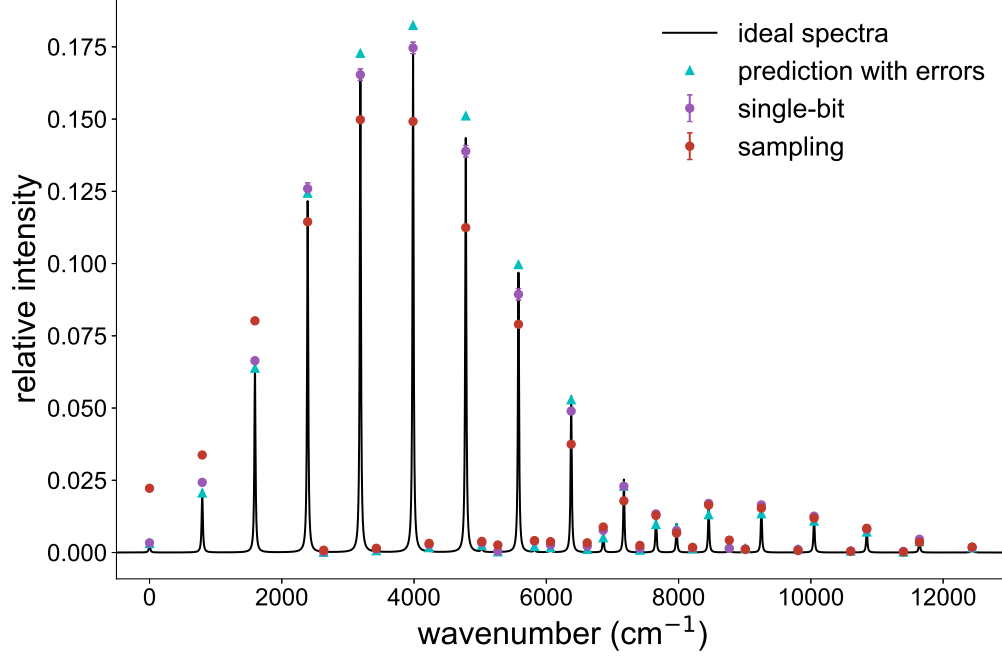


FIG. 5. Photoionization of nitrite to nitrogen dioxide starting in the vibrationless state $n = 0$, $m = 0$.

(n', m')	$\text{NO}_2 \xrightarrow{h\nu} \text{NO}_2 + e^-$ starting in $(n = 0, m = 0)$			
	Classically calculated	Master equation simulation	Single-bit extraction	Sampling
(0,0)	3.54E-03	3.14E-03	0.0033 ± 0.0004	0.0222 ± 0.0002
(0,1)	2.17E-02	2.07E-02	0.0243 ± 0.0008	0.0337 ± 0.0003
(0,2)	6.42E-02	6.40E-02	0.066 ± 0.001	0.0802 ± 0.0004
(0,3)	1.22E-01	1.24E-01	0.126 ± 0.002	0.1145 ± 0.0005
(0,4)	1.66E-01	1.73E-01	0.165 ± 0.002	0.1498 ± 0.0006
(0,5)	1.73E-01	1.82E-01	0.175 ± 0.002	0.1492 ± 0.0005
(0,6)	1.43E-01	1.51E-01	0.139 ± 0.002	0.1125 ± 0.0005
(0,7)	9.68E-02	9.95E-02	0.089 ± 0.002	0.079 ± 0.0004
(0,8)	5.41E-02	5.27E-02	0.049 ± 0.001	0.0375 ± 0.0003
(0,9)	2.53E-02	2.29E-02	0.0229 ± 0.0008	0.0179 ± 0.0002
(0,10)	9.93E-03	8.49E-03	0.0076 ± 0.0005	0.0067 ± 0.0001
(0,11)	3.29E-03	2.75E-03	0.0014 ± 0.0003	0.0043 ± 0.0001
(1,0)	2.74E-04	1.03E-04	0.00026 ± 0.0001	$0.0008 \pm 4\text{E-}05$
(1,1)	1.18E-03	6.52E-04	0.0012 ± 0.0002	$0.00147 \pm 6\text{E-}05$
(1,2)	2.24E-03	1.73E-03	0.0027 ± 0.0003	$0.00316 \pm 9\text{E-}05$
(1,3)	2.32E-03	2.48E-03	0.0033 ± 0.0003	0.00384 ± 0.0001
(1,4)	1.26E-03	2.04E-03	0.0039 ± 0.0004	0.00414 ± 0.0001
(1,5)	2.02E-04	1.04E-03	0.0022 ± 0.0003	$0.00336 \pm 9\text{E-}05$
(1,6)	6.79E-05	7.77E-04	0.0015 ± 0.0002	$0.00245 \pm 8\text{E-}05$
(1,7)	6.34E-04	1.27E-03	0.0015 ± 0.0002	$0.0018 \pm 7\text{E-}05$
(1,8)	1.12E-03	1.53E-03	0.0012 ± 0.0002	$0.00107 \pm 5\text{E-}05$
(1,9)	1.13E-03	1.15E-03	0.0011 ± 0.0002	$0.00067 \pm 4\text{E-}05$
(1,10)	8.04E-04	5.44E-04	0.0005 ± 0.0001	$0.00039 \pm 3\text{E-}05$
(1,11)	4.38E-04	1.56E-04	0.0004 ± 0.0001	$0.00021 \pm 2\text{E-}05$
(2,0)	4.71E-04	2.67E-04	0.0005 ± 0.0001	$0.0026 \pm 8\text{E-}05$
(2,1)	2.62E-03	1.71E-03	0.0026 ± 0.0003	$0.00376 \pm 9\text{E-}05$
(2,2)	7.06E-03	5.16E-03	0.0077 ± 0.0005	0.0088 ± 0.0001

(2,3)	1.24E-02	9.77E-03	0.0134 ± 0.0006	0.0128 ± 0.0002
(2,4)	1.58E-02	1.31E-02	0.017 ± 0.0007	0.0164 ± 0.0002
(2,5)	1.58E-02	1.35E-02	0.0165 ± 0.0007	0.0154 ± 0.0002
(2,6)	1.28E-02	1.09E-02	0.0126 ± 0.0006	0.0119 ± 0.0002
(2,7)	8.70E-03	7.12E-03	0.008 ± 0.0005	0.0084 ± 0.0001
(2,8)	5.07E-03	3.79E-03	0.0046 ± 0.0004	$0.00354 \pm 9E-05$
(2,9)	2.56E-03	1.67E-03	0.0017 ± 0.0002	$0.00191 \pm 7E-05$
(2,10)	1.14E-03	6.20E-04	0.0008 ± 0.0002	$0.00064 \pm 4E-05$
(2,11)	4.45E-04	1.96E-04	0.0005 ± 0.0001	$0.00039 \pm 3E-05$

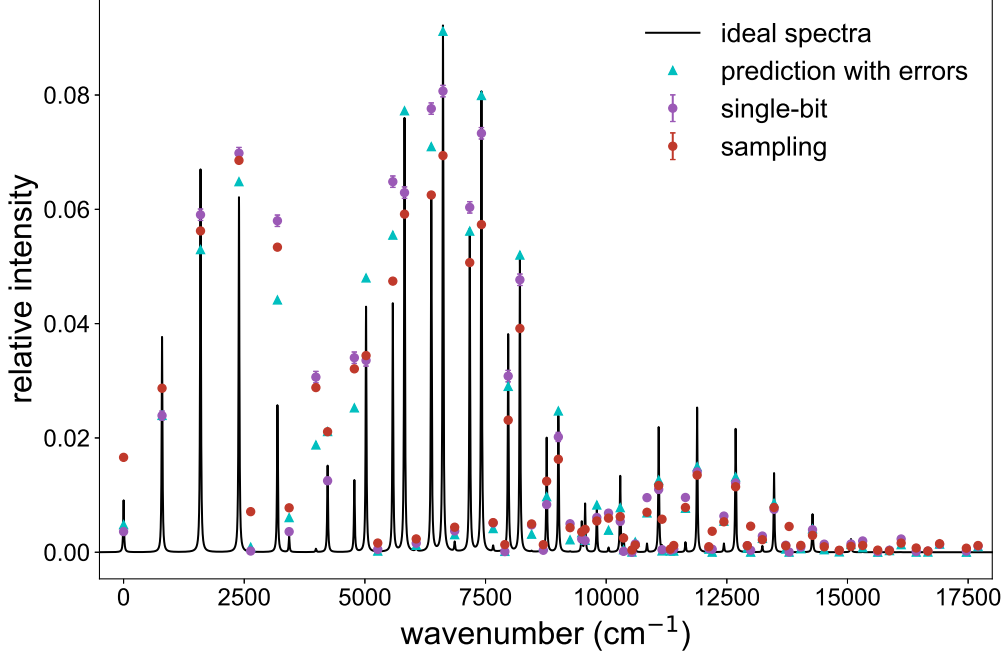


FIG. 6. Photoionization of nitrite to nitrogen dioxide starting with one quanta in the symmetric-stretching mode and zero in the bending mode $n = 1, m = 0$. The more significant errors are primarily due to having a large self-Kerr on cavity A (~ 30 kHz) during the beamsplitter operation after starting in a state with higher photon number.

$\text{NO}_2^- \xrightarrow{h\nu} \text{NO}_2 + e^-$ starting in $(n = 1, m = 0)$				
(n', m')	Classically calculated	Master equation simulation	Single-bit extraction	Sampling
(0,0)	9.08E-03	4.87E-03	0.0036 ± 0.0004	0.0166 ± 0.0002
(0,1)	3.77E-02	2.40E-02	0.0239 ± 0.0008	0.0287 ± 0.0003
(0,2)	6.70E-02	5.28E-02	0.059 ± 0.001	0.0562 ± 0.0004
(0,3)	6.21E-02	6.36E-02	0.07 ± 0.001	0.0686 ± 0.0004
(0,4)	2.57E-02	4.12E-02	0.058 ± 0.001	0.0534 ± 0.0004
(0,5)	5.78E-04	1.47E-02	0.0306 ± 0.001	0.0288 ± 0.0003
(0,6)	1.25E-02	2.20E-02	0.034 ± 0.001	0.0321 ± 0.0003
(0,7)	4.34E-02	5.43E-02	0.065 ± 0.001	0.0474 ± 0.0004
(0,8)	6.17E-02	7.15E-02	0.078 ± 0.001	0.0625 ± 0.0004
(0,9)	5.65E-02	5.71E-02	0.06 ± 0.001	0.0507 ± 0.0004
(0,10)	3.80E-02	2.95E-02	0.0308 ± 0.001	0.0231 ± 0.0003
(0,11)	2.00E-02	9.85E-03	0.0083 ± 0.0005	0.0124 ± 0.0002
(0,12)	8.41E-03	2.11E-03	0.002 ± 0.0003	0.004 ± 0.0001
(0,13)	2.87E-03	4.62E-04	0.0002 ± 0.0002	$0.00248 \pm 9E-05$
(0,14)	8.00E-04	2.41E-04	0.0004 ± 0.0002	0.0058 ± 0.0001
(1,0)	1.46E-04	9.10E-04	0.0002 ± 0.0002	0.0071 ± 0.0001
(1,1)	2.73E-03	6.17E-03	0.0036 ± 0.0004	0.0078 ± 0.0002
(1,2)	1.51E-02	2.15E-02	0.0125 ± 0.0006	0.0211 ± 0.0002
(1,3)	4.29E-02	4.89E-02	0.0336 ± 0.001	0.0344 ± 0.0003
(1,4)	7.59E-02	7.90E-02	0.063 ± 0.001	0.0592 ± 0.0004
(1,5)	9.21E-02	9.34E-02	0.081 ± 0.001	0.0694 ± 0.0004
(1,6)	8.05E-02	8.20E-02	0.073 ± 0.001	0.0573 ± 0.0004
(1,7)	5.16E-02	5.33E-02	0.048 ± 0.001	0.0392 ± 0.0003
(1,8)	2.39E-02	2.52E-02	0.0202 ± 0.0008	0.0163 ± 0.0002

(1,9)	7.45E-03	8.37E-03	0.0061 ± 0.0005	0.0055 ± 0.0001
(1,10)	1.24E-03	1.78E-03	0.0014 ± 0.0003	0.00124 ± 6E-05
(1,11)	1.95E-05	1.99E-04	0.0006 ± 0.0002	0.00118 ± 6E-05
(1,12)	8.94E-05	1.33E-05	0.0005 ± 0.0002	0.0037 ± 0.0001
(1,13)	1.56E-04	1.69E-05	0.0003 ± 0.0002	0.0046 ± 0.0001
(1,14)	1.08E-04	2.40E-05	0	0.0045 ± 0.0001
(2,0)	3.83E-04	1.99E-04	0.0005 ± 0.0001	0.00161 ± 7E-05
(2,1)	1.23E-03	1.20E-03	0.0016 ± 0.0002	0.00233 ± 8E-05
(2,2)	1.61E-03	3.06E-03	0.0037 ± 0.0004	0.0044 ± 0.0001
(2,3)	9.74E-04	4.06E-03	0.0051 ± 0.0004	0.0052 ± 0.0001
(2,4)	1.41E-04	2.92E-03	0.0048 ± 0.0004	0.005 ± 0.0001
(2,5)	1.05E-04	1.84E-03	0.005 ± 0.0004	0.0043 ± 0.0001
(2,6)	7.93E-04	3.63E-03	0.0068 ± 0.0005	0.006 ± 0.0001
(2,7)	1.49E-03	6.90E-03	0.0096 ± 0.0005	0.007 ± 0.0001
(2,8)	1.71E-03	7.86E-03	0.0096 ± 0.0005	0.0078 ± 0.0002
(2,9)	1.50E-03	5.59E-03	0.0063 ± 0.0004	0.0053 ± 0.0001
(2,10)	1.08E-03	2.48E-03	0.0028 ± 0.0003	0.00219 ± 8E-05
(2,11)	6.70E-04	6.10E-04	0.0009 ± 0.0002	0.0012 ± 6E-05
(2,12)	3.63E-04	5.24E-05	0.0003 ± 0.0001	0.00034 ± 3E-05
(2,13)	1.72E-04	1.32E-05	0.0001 ± 8E-05	0.00037 ± 3E-05
(2,14)	7.12E-05	2.40E-05	0.00014 ± 9E-05	0.00074 ± 5E-05
(3,0)	8.70E-05	1.50E-04	0.00012 ± 9E-05	0.00133 ± 6E-05
(3,1)	1.12E-03	9.95E-04	0.0003 ± 0.0001	0.00132 ± 6E-05
(3,2)	5.23E-03	3.46E-03	0.0023 ± 0.0003	0.0036 ± 0.0001
(3,3)	1.33E-02	7.92E-03	0.0054 ± 0.0004	0.0062 ± 0.0001
(3,4)	2.19E-02	1.29E-02	0.011 ± 0.0006	0.0117 ± 0.0002
(3,5)	2.53E-02	1.53E-02	0.014 ± 0.0007	0.0135 ± 0.0002
(3,6)	2.16E-02	1.35E-02	0.0121 ± 0.0006	0.0115 ± 0.0002
(3,7)	1.39E-02	8.77E-03	0.0075 ± 0.0005	0.0078 ± 0.0002
(3,8)	6.67E-03	4.15E-03	0.0039 ± 0.0004	0.00296 ± 9E-05
(3,9)	2.31E-03	1.37E-03	0.0014 ± 0.0002	0.00103 ± 6E-05
(3,10)	5.13E-04	2.86E-04	0.0004 ± 0.0001	0.00025 ± 3E-05
(3,11)	4.40E-05	3.18E-05	0.0002 ± 0.0001	0.00021 ± 2E-05
(3,12)	2.11E-06	7.28E-06	0.0002 ± 0.0001	0.00071 ± 5E-05
(3,13)	1.79E-05	8.77E-06	0.00018 ± 9E-05	0.00091 ± 5E-05
(3,14)	1.77E-05	7.78E-06	0.0001 ± 8E-05	0.00084 ± 5E-05
(4,0)	9.46E-06	3.58E-05	0	0.0004 ± 3E-05
(4,1)	8.74E-06	2.20E-04	0.0004 ± 0.0001	0.00055 ± 4E-05
(4,2)	2.89E-07	5.52E-04	0.0006 ± 0.0002	0.00096 ± 5E-05
(4,3)	3.10E-05	7.20E-04	0.0011 ± 0.0002	0.00115 ± 6E-05
(4,4)	8.57E-05	5.26E-04	0.0012 ± 0.0002	0.00112 ± 6E-05
(4,5)	9.82E-05	3.86E-04	0.0014 ± 0.0002	0.00095 ± 5E-05
(4,6)	6.28E-05	7.62E-04	0.002 ± 0.0003	0.00118 ± 6E-05
(4,7)	2.51E-05	1.35E-03	0.0023 ± 0.0003	0.0016 ± 7E-05
(4,8)	7.91E-06	1.48E-03	0.0014 ± 0.0002	0.00149 ± 7E-05
(4,9)	3.98E-06	1.03E-03	0.0011 ± 0.0002	0.00117 ± 6E-05
(4,10)	5.12E-06	4.58E-04	0.0004 ± 0.0001	0.00043 ± 4E-05
(4,11)	8.37E-06	1.20E-04	0.00015 ± 8E-05	0.00027 ± 3E-05
(4,12)	1.07E-05	1.67E-05	0	0.0001 ± 2E-05
(4,13)	1.00E-05	5.98E-06	0	7E-05 ± 1E-05
(4,14)	7.02E-06	5.32E-06	0.00015 ± 7E-05	0.00013 ± 2E-05

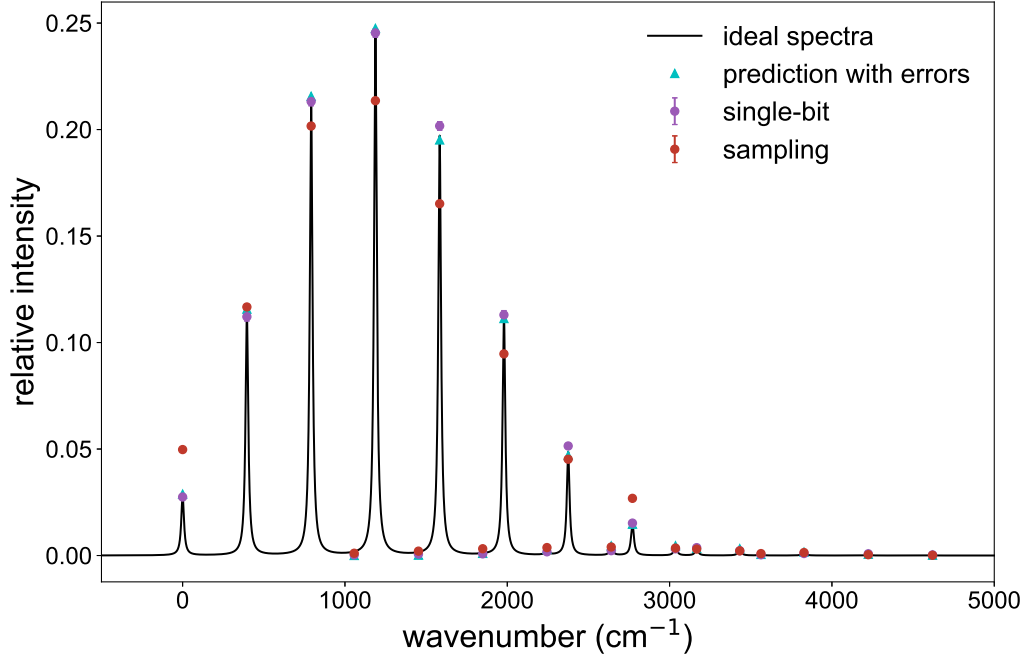


FIG. 7. Photoionization of sulfur dioxide to the cation starting in the vibrationless state $n = 0$, $m = 0$.

(n', m')	$\text{SO}_2 \xrightarrow{h\nu} \text{SO}_2^+ + e^-$ starting in $(n = 0, m = 0)$			
	Classically calculated	Master equation simulation	Single-bit extraction	Sampling
(0,0)	2.82E-02	2.88E-02	0.0275 ± 0.0009	0.0497 ± 0.0003
(0,1)	1.14E-01	1.15E-01	0.112 ± 0.002	0.1167 ± 0.0005
(0,2)	2.13E-01	2.15E-01	0.213 ± 0.002	0.2017 ± 0.0006
(0,3)	2.47E-01	2.47E-01	0.245 ± 0.002	0.2136 ± 0.0006
(0,4)	1.97E-01	1.95E-01	0.202 ± 0.002	0.1652 ± 0.0006
(0,5)	1.14E-01	1.11E-01	0.113 ± 0.002	0.0947 ± 0.0005
(0,6)	4.85E-02	4.72E-02	0.051 ± 0.001	0.0452 ± 0.0003
(0,7)	1.54E-02	1.48E-02	0.0152 ± 0.0007	0.0268 ± 0.0003
(0,8)	3.57E-03	3.38E-03	0.0037 ± 0.0004	$0.00302 \pm 9E-05$
(0,9)	5.68E-04	5.28E-04	0.0003 ± 0.0002	$0.00091 \pm 5E-05$
(1,0)	9.40E-06	2.89E-05	0.0003 ± 0.0001	$0.00108 \pm 5E-05$
(1,1)	4.35E-05	1.53E-04	0.0007 ± 0.0002	$0.00206 \pm 7E-05$
(1,2)	7.70E-04	9.82E-04	0.0008 ± 0.0002	$0.00318 \pm 9E-05$
(1,3)	2.67E-03	2.84E-03	0.0017 ± 0.0003	0.00374 ± 0.0001
(1,4)	4.54E-03	4.53E-03	0.0023 ± 0.0003	0.004 ± 0.0001
(1,5)	4.78E-03	4.60E-03	0.0031 ± 0.0003	$0.0035 \pm 9E-05$
(1,6)	3.44E-03	3.22E-03	0.0022 ± 0.0003	$0.00219 \pm 7E-05$
(1,7)	1.76E-03	1.62E-03	0.0009 ± 0.0002	$0.0014 \pm 6E-05$
(1,8)	6.53E-04	5.89E-04	0.0008 ± 0.0002	$0.00038 \pm 3E-05$
(1,9)	1.73E-04	1.54E-04	$0.0003 \pm 9E-05$	$0.00011 \pm 2E-05$

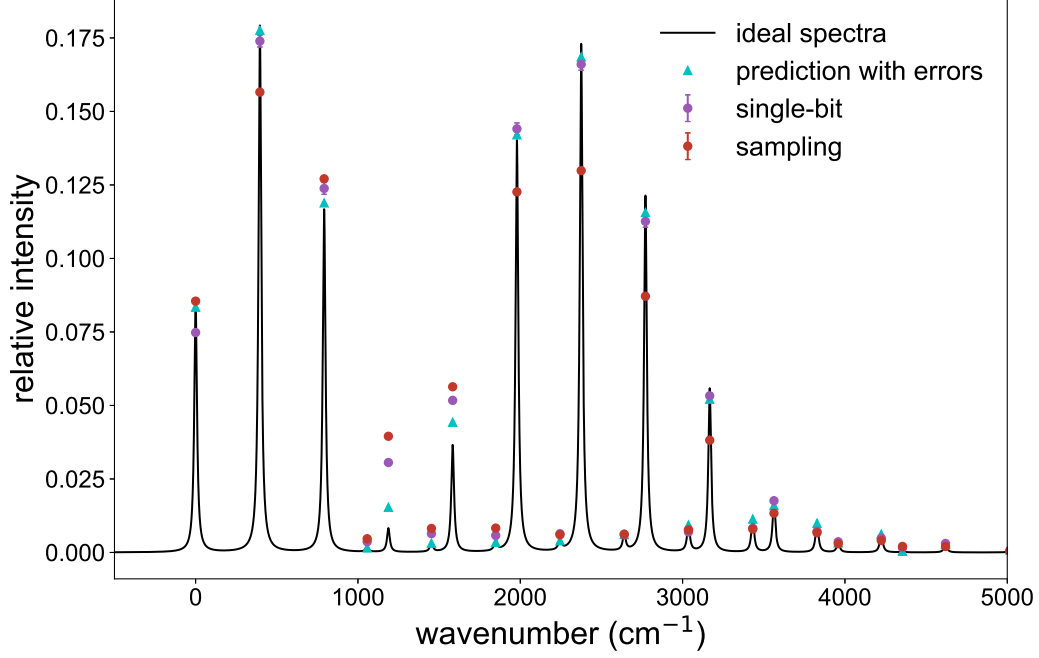


FIG. 8. Photoionization of sulfur dioxide to the cation starting with zero quanta in the symmetric-stretching mode and one quantum in the bending mode $n = 0$, $m = 1$.

$\text{SO}_2 \xrightarrow{h\nu} \text{SO}_2^+ + e^-$ starting in $(n = 0, m = 1)$				
(n', m')	Classically calculated	Master equation simulation	Single-bit extraction	Sampling
(0,0)	8.52E-02	8.36E-02	0.075 ± 0.001	0.0855 ± 0.0005
(0,1)	1.79E-01	1.78E-01	0.174 ± 0.002	0.1566 ± 0.0006
(0,2)	1.17E-01	1.19E-01	0.124 ± 0.002	0.1271 ± 0.0006
(0,3)	8.11E-03	1.50E-02	0.0306 ± 0.0009	0.0395 ± 0.0003
(0,4)	3.64E-02	4.42E-02	0.052 ± 0.001	0.0564 ± 0.0004
(0,5)	1.40E-01	1.42E-01	0.144 ± 0.002	0.1226 ± 0.0006
(0,6)	1.73E-01	1.69E-01	0.166 ± 0.002	0.1299 ± 0.0006
(0,7)	1.21E-01	1.16E-01	0.113 ± 0.002	0.0872 ± 0.0005
(0,8)	5.56E-02	5.22E-02	0.053 ± 0.001	0.0382 ± 0.0003
(0,9)	1.74E-02	1.60E-02	0.0176 ± 0.0007	0.0133 ± 0.0002
(0,10)	3.62E-03	3.24E-03	0.0036 ± 0.0004	$0.00296 \pm 9E-05$
(0,11)	4.51E-04	3.96E-04	0.0016 ± 0.0003	$0.00206 \pm 8E-05$
(1,0)	1.68E-03	1.55E-03	0.0037 ± 0.0003	0.0047 ± 0.0001
(1,1)	3.27E-03	3.16E-03	0.0064 ± 0.0005	0.0082 ± 0.0002
(1,2)	3.48E-03	3.48E-03	0.0058 ± 0.0004	0.0083 ± 0.0002
(1,3)	3.89E-03	3.99E-03	0.0064 ± 0.0004	0.006 ± 0.0001
(1,4)	5.95E-03	6.09E-03	0.006 ± 0.0004	0.0062 ± 0.0001
(1,5)	9.44E-03	9.38E-03	0.007 ± 0.0005	0.0077 ± 0.0001
(1,6)	1.18E-02	1.14E-02	0.0083 ± 0.0005	0.008 ± 0.0001
(1,7)	1.07E-02	9.95E-03	0.0072 ± 0.0005	0.0067 ± 0.0001
(1,8)	6.90E-03	6.24E-03	0.0048 ± 0.0004	0.0041 ± 0.0001
(1,9)	3.18E-03	2.79E-03	0.0031 ± 0.0003	$0.00203 \pm 8E-05$
(1,10)	1.03E-03	8.79E-04	0.0005 ± 0.0001	$0.00061 \pm 4E-05$
(1,11)	2.26E-04	1.86E-04	$4E-05 \pm 6E-05$	$0.00035 \pm 3E-05$

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